Heterogeneous Agents in Macro Models

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Problem set 4

Solving the Hugget model in continuous time

Model

Consider a model where there is no aggregate risk and no aggregate dynamics. Supply of capital is fixed at some exogenous level S.

There is idiosyncratic risk in the economy. Individuals can either be "employmed" or "unemployed". When employed, they obtain an endowment y_2 and they obtain an endowment y_1 when unemployed, where $y_2 > y_1$. Individuals cannot, however, hold capital holdings less than the limit $\underline{k}$.

An employed individual becomes unemployed at rate λ_e and an unemployed individual becomes employed at rate λ_u. The dynamics of (un)employment can then be written as

$$\dot{e}_t = -\lambda_e e_t + \lambda_u u_t$$

$$\dot{u}_t = \lambda_e e_t - \lambda_u u_t$$

Dynamics of the distribution can then be summarized as

$$\dot{g}_t = T g_t,$$

where $T = \begin{pmatrix} -\lambda_e & \lambda_u \\ \lambda_e & -\lambda_u \end{pmatrix}$. The stationary distribution is then given by

$$0 = Ts$$

Therefore, s is an eigenvector associated with a zero eigenvalue (and normalized to

1).[1]

The HJB equations can be written as

$$\rho v_e(\mathbf{a}) = \max_c \left[u(\mathbf{c}) + v_e'(\mathbf{a})(y_2 + r\mathbf{a} - \mathbf{c}) - \lambda_e(v_e(\mathbf{a}) - v_u(\mathbf{a}))\right] \tag{1}$$

$$\rho v_u(\mathbf{a}) = \max_c \left[u(\mathbf{c}) + v_u'(\mathbf{a})(y_1 + r\mathbf{a} - \mathbf{c}) - \lambda_u(v_u(\mathbf{a}) - v_e(\mathbf{a}))\right] \tag{2}$$

Question 1

Read the code in "AiyagariContmain.m" carefully and make sure you understand its structure.

(i) complete the gaps in "AiyagariContmain.".

Question 2

(i) Once you're done, run the program. How fast is it? (Hint: is it amazingly fast?!)

(ii) Plot the distribution of asset holdings for the employed and unemployed.

(iii) Plot the policy rules (savings and consumption) for the employed and the unemployed.

[1]But, eigenvector is defined only up to a scalar (will need a trick in the code).